An exemplary quotient

As a lead-in to our extension of the number theory of $\mathbb{Z}$ to polynomial rings $R[x]$, let's consider the following problems, in which $R = \mathbb{Z}$.

1. Let $g(x) = 3x^5 + 7x^3 + 7x^2 - 19x + 16$ and $f(x) = x^2 + 4$. Use polynomial long division to find the quotient $\frac{g}{f}$.

2. What can you say about the remainder, $r(x)$, stemming from the above computation?

3. Compute the polynomial $g(x) - q(x)f(x)$, where $q(x)$ is the quotient you obtained in (1). Coincidence?

The *leading term* of a polynomial $p(x) = s_0 + s_1 x + \cdots + s_n x^n$ (where $s_n \neq 0$) is the term $s_n x^n$. We'll denote the leading term of p by $LT(p)$, as your textbook does.

4. Now let's consider different polynomials, say $\rho(x) = 5x^3 - x^2 + 1$ and $\phi(x) = x^2 + x + 2$. What are $LT(\rho)$ and $LT(\phi)$?

5. Compute $\rho(x) - \frac{LT(\rho)}{LT(\phi)}\phi(x)$.

6. Describe qualitatively the effect this last operation has on the leading term of ρ.