Tools of the Trade: Notation and Definitions, volume IV

More!

(1) Let's start with some more set theory notation and terminology:
- The *set difference* of S and T is the set $S \setminus T = \{s \in S \mid s \notin T\}$.
- In particular, if $A \subseteq S$, then the *complement of* A *in* S, denoted by A^c (where S is implicitly understood), is $A^c = S \setminus A = \{s \in S \mid s \notin A\}$.
- Remember we proved *DeMorgan Laws* for complementation, that worked kind of like logical negation; for any sets A and B in the powerset $\mathcal{P}(S)$, we have
$$(A \cup B)^c = A^c \cap B^c \quad \text{and} \quad (A \cap B)^c = A^c \cup B^c.$$
- The *Cartesian product* $S \times T$ of sets S and T is the set
$$\{(s,t) \mid s \in S \text{ and } t \in T\}$$
of *ordered pairs*.

(2) We continue with some basic combinatorial principles:
- The *Pigeonhole Principle*, in its most general form says that if $nk+1$ or more objects are placed into n boxes, then some box contains at least $k+1$ objects.
- Recall that a *partition* of a set S is a set $\{S_i\}$ of *disjoint* nonempty subsets of S such that $\bigcup_i S_i = S$. The *Addition Principle* says that if $\{S_i\}$ is a partition of S, then
$$|S| = \sum_i |S_i|.$$
More generally, we guessed in class that
$$\left|\bigcup_i S_i\right| \leq \sum_i |S_i|,$$
since if the sets S_i are not disjoint, we might have overlap between them that the sum counts twice.
- Meanwhile, the *Multiplication Principle* tells us that if $S_1, S_2, ..., S_n$ are sets, then
$$|S_1 \times S_2 \times \cdots \times S_n| = |S_1| \cdot |S_2| \cdot \;\cdots\; \cdot |S_n|.$$

(3) Our foray into combinatorics continued with a discussion of combinations. Given a set S, a *k-combination* in S is a subset of S of size k.
Theorem. If $|S| = n$, the number of k-combinations in S is given by $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.

(4) Finally, we discussed permutations. Given a set S, a *k-permutation* in S is an ordered list of k distinct elements of S.
Theorem. If $|S| = n$, the number of k-permutations in S is given by $\frac{n!}{(n-k)!}$.

Our next episode of *Tools of the Trade* will continue after we've discussed *relations*!