MATH 280, Section 1
Foundations of Mathematics
Spring 2009

Tools of the Trade: Notation, volume I

The is the first in a series of notation lists with which I will provide you as the semester progresses. Compiled below is a collection of mathematical symbols you will likely find useful in reading and writing in mathematics. Unlike a number of symbols we'll use for different meanings at different times (like x, S, *etc.*), each of these symbols will have a single, fixed meaning for the duration of our class, and in most cases beyond.

I encourage you to use these symbols whenever appropriate, in order to get used to their proper application.

(1) **Common sets.** The following mathematical sets are important enough to have notation set aside for them.
   (a) $\mathbb{N}$ is the set of *natural numbers*, $\{1, 2, 3, ...\}$. If we want to include 0, we may write $\mathbb{N}_0$.
   (b) $\mathbb{Z}$ is the set of *integers*, $\{..., -2, -1, 0, 1, 2, ...\}$.
   (c) $\mathbb{Q}$ is the set of *rational numbers* (fractions), $\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$.
   (d) $\mathbb{R}$ is the set of *real numbers*, which for the time being we may think of as all "decimal numbers" (terminating or not, repeating or not).
   (e) $\mathbb{C}$ is the set of *complex numbers*, $\{\alpha + \beta i \mid \alpha, \beta \in \mathbb{R}, i = \sqrt{-1}\}$.
   (f) $\mathbb{P}$ is the set of *prime numbers*, integers $p \geq 2$ for which 1 and p are the only positive integers dividing p evenly: $\mathbb{P} = \{2, 3, 5, 7, 11, ...\}$. (We will prove later that there are infinitely many primes.)

(2) **Logical symbols.** The following are symbols that come up frequently in basic propositional logic.
   (a) $\forall$ is the *universal quantifier*, typically read "for all" or "for every."
   (b) $\exists$ is the *existential quantifier*, typically read "there exists" or "for some."
   (c) $\wedge$ is the symbol for logical *conjunction*, expressing the logical sense of the word "and." Thus $P \wedge Q$ is read "P and Q."
   (d) $\vee$ is the symbol for logical *disjunction*, expressing the logical sense of the word "or." Thus $P \vee Q$ is read "P or Q."
   (e) $\Leftrightarrow$ signifies the *biconditional* relation, expressing the logical sense of the words "if and only if." Thus $P \Leftrightarrow Q$ is read "P [is true] if and only if Q [is true]."
   (f) $\Rightarrow$ signifies the *conditional*, or *implication*, relation, expressing the logical sense of the word "implies." Thus $P \Rightarrow Q$ is read "P implies Q" or "if P [is true], then Q [is true]."