MATH 280, Section 1
Foundations of Mathematics
Spring 2009

Syllabus

Let's take care of some of the bureaucratic matters first:

- **Who's the teacher?** Dr. Patrick Bahls (please feel free to call me Patrick).
- **When is class?** 12:45 p.m. to 1:35 p.m. on Monday, Wednesday, and Friday.
- **Where is class?** Carmichael Hall, Room 132: lots of open space and room to write!
- **What text are we using?** *How to solve it* (Expanded Princeton Science Library Edition), by George Polya. (ISBN 0-691-11966-X.) This is not a textbook in the traditional sense; rather, readings will be assigned from it, and we will sometimes use it to launch discussions in class.
- **What do I bring to class?** Yourself, mind intact. Also, something with which, and something *upon* which, to write.
- **What about office hours?** Most of you know that I am available roughly 20 to 25 hours throughout the week: if my door's open, you've caught me. Soon I will post and announce hours at which you will be *guaranteed* to find me in my office, and you should feel free to make an appointment with me if you'd like to be sure of securing my time.
- **How can I get a hold of you?** My office phone number is 232-5190, and my e-mail address is patrick.bahls@gmail.com. (E-mail's the best way to get me.) Or just stop on by my office; if you catch me, I'm yours: Room 324, Robinson Hall.
- **Do you have a website?** Yes indeed: http://facstaff.unca.edu/pbahls. From there you can find easy links to the course websites, and resources like this here syllabus. Chances are if you'll need anything course-related throughout the semester, it'll be on the website, so please look before asking.
- **What other resources will I have available to me?** As I'm sure many of you know, the Math Lab, located across the hall from my office in Robinson Hall, is an *excellent* place to meet and greet your friends from off the street. Please consider gathering there to do your homework, to get tips from the dependable Math Lab staff, and to get help from me when you're stuck.

What will we be studying? The primary purpose of this course is to introduce you to the notion of *mathematical proof*. To prove a statement is to do more than to simply convince someone of the truth of the statement; you must moreover demonstrate by means of a mathematical argument that the statement you make is *a logically necessary conclusion* derived by means of accepted rules of

deduction from hypotheses you have set forth to begin with. Coming up with such a demonstration requires a firm understanding of the basic rules of mathematical logic, the essence of good notation and terminology, and a grasp of ideas like *hypothesis*, *conclusion*, and *proof*.

Secondarily, we will be learning a good deal about certain fields within mathematics. After all, in order to learn how to prove something, we must have something to prove! The "somethings" we prove will be drawn primarily from elementary set theory and combinatorics, number theory, and other topics from foundational mathematics. The following is a rough list of the topics we will be discussing, in the order in which we will consider them:

- basic propositional logic and "truth,"
- methods of proof, including induction,
- the basic theory of sets,
- combinatorics, including combinations and permutations,
- the formal definition of a function,
- cardinality of sets,
- relations in general,
- equivalence relations and order relations in particular, and
- axiomatic systems, illustrated by groups.

As I mentioned briefly above, the assigned text for this course is not a textbook in the ordinary sense. When you look through it (you will be asked to do this, through various reading assignments), you will notice that there are relatively few mathematical facts or statements made in its pages. Instead, there are general outlines of arguments, explanations of methods of proof. Your text may often be helpful in providing you with hints in solving the math problems we encounter, as it will help you focus on the most general, and in many ways the most important, mathematical principles you will ever encounter.

What do you expect me to get out of this course? Think of the following list as a set of skills I'd love for you to have five years after taking this course:

- Be able to give both rigorous and intuitive definitions of *proof*, *hypothesis*, and *conclusion*.
- Be able to read and explain a formal mathematical proof you yourself did not construct.
- Approach a new piece of mathematical writing with confidence.
- Understand and demonstrate a few basic problem-solving and proof techniques one might use to prove an unfamiliar mathematical statement.
- Be able to assess various mathematical statements for their relative strengths and weaknesses.

- Appreciate the importance of clarity in a mathematical argument, as well as correctness.
- Develop the ability to construct rough arguments and estimates as well as precise proofs and exact figures.

It might not hurt you to go back over this list every so often and ask yourself (and me!), "am I making progress in developing these skills?" If at any time you feel the answer is "no," please come and talk to me about it; one or both of us might be able to change our ways to help you better reach these goals.

What kind of work will I do in this course? Your grade will be based upon completion, peer-review, and discussion of homework problems; facility with the mathematical typesetting software LaTeX; three take-home exams scattered throughout the semester; and an end-of-semester presentation on a proof-laden topic chosen from a textbook or undergraduate mathematics journal.

- *Homework.* Homework will be assigned roughly once per week and will typically consist of several exercises taken from various textbooks and journal articles, or made up by me. All homework will be graded for correctness, completeness, and clarity, and graded work will be returned to you promptly.

  In addition to completing the homework, you will also be asked to take part in peer review and class discussions on selected homework problems. Furthermore, you will be required to produce your solutions to certain problems using LaTeX typesetting software.

  Here is more information about these last two points.

  Homework committees. From each homework set, one or two "committee problems" will be selected as the focus of peer-review. For each such exercise, I will form a committee of three of you to meet and discuss all student solutions to the given problem. For your solutions to be considered by this committee, they will have to have been submitted by a certain date, well before the homework assignment's due date. In order to receive credit for participating in the peer-review process, you must submit solutions to at least half of all committee problems to the respective committee during the course of the semester.

  The committee for a given problem will meet outside of class to discuss the solutions that have been submitted to them. They will then offer feedback to each solution's author, checking the solution for correctness, completeness, and clarity, and indicating strengths and weaknesses of the solution. It is the responsibility of each committee to verify a solution's quality. Although I will be available for consultation should the need arise, I feel very strongly that *you too must develop the authority to assess the quality of mathematical arguments.*

  The committee will write a brief (one- or two-paragraph) report on their overall impressions of the solutions submitted to them, and will provide me with a list of their peers they feel made a substantial effort in completing a first draft of a solution. Finally, the committee will lead a brief discussion on the assigned problem with the class, highlighting the strengths and weaknesses of the solutions submitted. (It is *not* the committee's job to provide a correct solution in their discussion!) After this discussion, the completed solutions will be returned

to their authors, who will be given the opportunity to make corrections or adjustments as needed before returning a final draft of the solution to me.

A typical weekly schedule might look like this:

- **1st Monday:** Homework is first assigned.
- **Wednesday:** Submissions are due to the committee.
- **Friday:** Committee leads discussion.
- **2nd Monday:** Homework, with revisions, is due.

Homework completion will count for 40% of your grade, and active participation in the peer review (both as a submitter and as a reviewer) and class discussions will count for 10%.

LaTeX *typesetting software.* Ever wonder how it is that I manage to make such snazzy-looking and easy-to-read mathematical documents, containing everything from $e^{e^{e^x}}$ and $\int_a^b \sin(x)\,dx = \cos(a) - \cos(b)$ to $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ and $\langle a, b \mid aba^{-1}b^{-1} \rangle \cong \mathbb{Z} \times \mathbb{Z}$? The ticket to this typographical territory is called "LaTeX," a programming language of sorts consisting of typesetting commands that tell the software that compiles it how to make up mathematical formulas, formatting commands, and much more.

LaTeX is an absolutely invaluable tool in mathematics: it gives the mathematician manipulative power that in olden days was solely the purview of a publishing company's trained typesetter. Much more importantly, its use demands that the mathematician organize her thoughts in a very straightforward fashion: typeset words don't contain marginalia, arrows pointing to extraneous commentary, or "oopsies!" afterthoughts that detract from the main thrust of an argument. That is, in being asked to type one's ideas rather than simply write them, one is being asked to thoroughly think one's answers through.

In the first few weeks of class you will be introduced to the LaTeX environment and will be asked to obtain the software necessary to use LaTeX on your own computer (all that you'll need is available for free online). Beginning with the second homework assignment, an increasingly large percentage of homework problems will be designated as "LaTeX problems" to which your final answers must be typeset using the LaTeX software. You will receive a small amount of extra credit for producing LaTeX solutions for other problems as well.

Don't worry, LaTeX is not difficult to learn and incredibly fun to use! I will give you plenty of resources to get you started, and I will be available all semester long to help you if you get stuck on a tricky typographic detail.

- *Exams.* There will be three exams, each worth 15% of your final grade. They will be take-home exams, given roughly in the 5th week, the 10th week, and the 14th week of class. I will post precise dates on the course website as these dates approach. Make-up exams will be allowed only in case of excused absences arranged well in advance, and I reserve the right not to allow a make-up exam.

  Much like the homework, each exam will be completed in two phases. After you have submitted your exam, I will read through it, proffer a tentative grade, and provide you with

feedback on your solutions. After I have done this, the exams will be returned to you, and you will be allowed to perform revisions of your solutions in order to improve your proofs before resubmitting the final version. Your grade on each exam will be computed by adding 1/4 of the points missed on each problem in your preliminary draft for successful improvements in the final draft. That is, if you miss 12 points on a particular problem in the preliminary version, but you nail the solution in revision, you'll receive an additional $12/4 = 3$ points above the score you got for that problem in the first draft.

- *End-of-semester presentations.* As important as learning how to communicate mathematics through writing is learning how to communicate mathematics through speech. Therefore, in addition to the homework discussions you will all take part in, in the final week or two of class each of you, working in pairs, will be asked to give a 10- to 12-minute presentation on a technical and proof-heavy mathematical topic we have not covered during the preceding portion of the semester. You will be asked to select topics for these presentations by the 10th week of class so that you will have plenty of time to prepare your presentation. I will be happy to you assist you in your preparation in any way (topic selection, dry runs, technical details, *etc.*). Your presentation will be worth 5% of your overall grade.

This course is "Writing-Intensive." What does that mean? Yes, this course now comes with a built-in WI rating. Why? Simply because WI courses are those in which "writing in the discipline" plays a major role, and at the very heart of the matter, MATH 280 is meant to teach you *how to write mathematically*. That's the point of the class! Through in-class exercises and homework problems, you will learn that doing math well is more than simply performing the correct computations or getting the right numbers at the end of the day. Doing math involves *clear* and *cohesive* communication of your mathematical ideas: a mathematician fails if she has brilliant ideas that she is unable to share effectively with others.

Central to the mathematical discipline is the expression of math in writing. We will take pains to understand how to properly write a mathematical proof, in essence learning the "grammar" and "syntax" of math writing. We will learn how writing in mathematics, like writing in any other discipline, is an incremental process: beautiful proofs don't just appear on the page, they are built up, step by step, revision after revision, the result of one's peers' insights as well as your own. Throughout the semester we will take part in exercises meant to strengthen your mathematical expression, to help you understand the ingredients of clear exposition, and to learn how to assess the quality of others' writing. You will find that throughout all of this LaTeX plays a crucial role, due to its global prevalence in mathematical communication.

That's all that I have to say about our class for now. Please don't hesitate to let me know at any time if you have questions, or if concerns arise. Let's go ahead and dive right in!