

Greatest Hits, volume 3

I've selected one each from among the solutions to *different* problems for this week's installment of MATH 280's *Greatest Hits*.

This first one is distinguished by its clarity and composition: it's among the shortest of the proofs submitted, but astute choice of wording and phrasing leaves nothing out. Note that even though the second case isn't proven, that the author of this proof recognized the *need* for a second case is the crucial fact: as we mentioned in class the other day, the appeal to an "analogous" proof is perfectly fine if there's really nothing substantively different in the second proof.

(2) Theorem: The difference between two consecutive cubes is odd.

Proof: If $n \in \mathbb{Z}$, then we have two cases to consider:

Case I: The larger consecutive cube is odd.

Case II: The larger consecutive cube is even.

Case I:

$$\begin{aligned}(2n+1)^3 - (2n)^3 &= 8n^3 + 12n^2 + 6n + 1 - 8n^3 \\ &= 2(6n^2 + 3n) + 1\end{aligned}$$

Let $6n^2 + 3n = k \in \mathbb{Z}$, then

$$(2n+1)^3 - (2n)^3 = 2k + 1, \text{ which is an odd number.}$$

Case II: Is analogous to Case I.

This week's second proof is one of the finest responses to 5.b, the tricky committee problem in which one must be *sure* to prove *both* implications. That is, one must show both that if n is prime then $\sigma(n) = n + 1$, *and* that if $\sigma(n) = n + 1$, then n is prime. Let's see how our resident expert managed this:

- $\sigma(n) = n + 1$ if and only if n is prime

Proof. Assume some natural number n is prime. That is to say that n 's only divisors are n and 1. Thus $\sigma(n) = n + 1$.

Now assume n to still be a natural number, but not prime. We know that it still has divisors n and 1, but by definition it must have other positive, non-zero divisors. If we define the sum of these divisors to be k , we can write $\sigma(n) = n + 1 + k$ showing that $\sigma(n) = n + 1$ if and only if n is prime

By the way, this solution was impeccably L^AT_EXed, to boot!