MATH 280, Section 1
Foundations of Mathematics
Spring 2009

Greatest Hits, volume 2

The following response was hands-down the strongest one received to Exercise 4 on the second homework set. There are a couple of minor slips, but no mistakes of any real consequence. Note how the response has strong composition, with every line in its logical place. This composition makes the work eminently clear. Moreover, the answer is correct in every detail, and completely answers the question posed.

a.) $\neg Q$ would mean "$x + y$ is even."

b.) P is "x is odd and y is even" or better yet P_1 = "x is odd" and P_2 = "y is even." DeMorgan's Law states that $P_1 \wedge P_2 \Leftrightarrow \neg P_1 \vee \neg P_2$. Therefore $\neg P_1$ = "x is even." $\neg P_2$ = "y is odd."
Thus, $\neg P = x$ is even or y is odd.

c.) By contraposition, assume "$x + y$" is even.
Thus, $x + y = 2n$, where n is some integer.
Suppose that y is even.
Thus, $x + 2m = 2n$ where m is also some other integer.
Therefore $x = 2n - 2m$, or $x = 2(n - m)$ where $n - m$ = some integer because one integer minus another integer equals an integer. So, $x = 2(k)$ where k is some integer.
Therefore 2 times some integer is even by definition.
Therefore, x is even.