MATH 280, Section 1
Foundations of Mathematics
Spring 2009

Exam 3

Working by yourself, please answer each question below on your own paper. You may use your class notes (but please no other resources), and you may consult with me (but I promise not to be too helpful). You may get help neither from each other nor from Math Lab staff nor from any other faculty or staff at UNC Asheville.

The exam is worth a total of 50 points, and the point value of each question is given with that question. Your exam is due at 5:00 p.m. on **Friday, May 8th**. Late exams will not be accepted, so please be sure to complete your exam on time, and please, please, ***please*** see me if you have any questions!

(1) (10 points) Let q be the relation on $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b) \sim_q (c, d) \Leftrightarrow ad = bc.$$

(For those who like to see their relations given as pairs, the above formula translates to $\big((a, b), (c, d)\big) \in q \Leftrightarrow ad = bc$.) Prove that q is an equivalence relation.

Note: we considered this relation as it applied to $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ in the last days of class, using it to define the rational numbers $\mathbb{Q}$. Here for simplicity we ignore integers ≤ 0.

(2) (10 points) Prove that the set of functions from $\mathbb{N}$ to itself is uncountable.

Hints: the set $\mathbb{N}^{\mathbb{N}}$ of functions from $\mathbb{N}$ to $\mathbb{N}$ obviously contains the set $[2]^{\mathbb{N}}$ of functions from $\mathbb{N}$ to $\{0, 1\}$. According to Homework Problem 8.2, what set has the same cardinality as $[2]^{\mathbb{N}}$? What does our class's very last theorem have to say about all of this? Try to put these ideas together to get your proof!

(3) (10 points total; 5 points each) Let C be the set of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

(a) Define $\leq_1$ on C by

$$f \leq_1 g \Leftrightarrow f(0) \leq g(0).$$

Prove that $\leq_1$ is *not* an order relation on C. State which of the defining conditions of an order relation $\leq_1$ does satisfy, and prove your assertion.

(b) Define $\leq_2$ on C by

$$f \leq_2 g \Leftrightarrow (\forall x \in \mathbb{R})\ f(x) \leq g(x).$$

Is $\leq_2$ an order relation on C? Prove your assertion.

(4) (10 points) Compute the number of possible order relations that can be defined on the set $\{a, b, c, d\}$.

Hints: first find the number of "types" of Hasse diagrams you can form without putting names on the points in the diagrams. Then use basic combinatorics (Addition Principle, Multiplication Principle, *etc.*) to figure out how many different relations correspond each "type" of diagram, keeping in mind that you'll get a different number for different types. Remember that in a Hasse diagram one doesn't draw a line from a to c if both $a \leq b$ and $b \leq c$ and there are already lines from a to b and from b to c.)

(5) (10 points) Please compose a brief statement detailing any advice you would like to give to someone taking 280, to be given to the student on the first day of class. You can be as specific or as general as you like, but please be sure that your response offers honest and thoughtful advice to a new 280 student.