

You Can Count on It, III:

Permutations

To end our brief discussion on counting, I'd like to introduce one more concept, this one closely related to combinations.

Definition. Given a set S , a k -permutation in the set S is an *ordered list* (s_1, s_2, \dots, s_k) of k elements chosen from S .

The important difference between a k -combination and a k -permutation is that the order doesn't matter in the case of a combination: $\{1, 2, 3\}$ and $\{3, 1, 2\}$ both represent the same combination, whereas $(1, 2, 3)$ and $(3, 1, 2)$ are *different* permutations.

Example. Find all k -permutations of the set $\{1, \pi, e\}$, for $0 \leq k \leq 3$.

Now that we've got a formula for the number of k -combinations in a set S with n elements, it's a simple matter to find the number of k -permutations. The key is the following observation:

Lemma. Let $|S| = n$ and suppose $0 \leq k \leq n$. To every k -combination $\{s_1, \dots, s_k\}$ in S , there correspond precisely $k!$ k -permutations. Moreover, every k -permutation corresponds to exactly *one* k -combination.

Another way of saying this is that there is a " $k!$ -to-1" function taking permutations to combinations. This is an idea we'll return to once we have a more precise formulation of functions.

