MATH 280, Section 1
Foundations of Mathematics
Fall 2009

Greatest Hits, Vol. II: Homework 3.3

3. Prove that $\sqrt{2}$ is irrational.

We will prove $\sqrt{2}$ is irrational by contradiction. Assume by way of contradiction that $\sqrt{2} = p/q$ where $p, q \in \mathbb{Z}$ and $\frac{p}{q}$ is written in lowest terms, then

$$\sqrt{2} = \frac{p}{q} \Rightarrow 2 = \frac{p^2}{q^2} \Rightarrow 2q^2 = p^2.$$

Since q^2 is some number, and p^2 is twice q^2, p^2 must be even, and since only an even number times and even number yields an even number, p must be even.

But if p is even it can be written as $2n$, so

$$2 = \frac{(2n)^2}{q^2} \Rightarrow 2q^2 = 4n^2 \Rightarrow q^2 = 2n^2.$$

So q^2 is twice n^2, which makes q^2 even, which implies that q is also even.

If p and q are both even, then p and q are divisible by two, therefor p/q is not in lowest terms, therefor we have a contradiction.