MATH 280, Section 1
Foundations of Mathematics
Fall 2009

Greatest Hits, Volume I: Homework 1.3

3. Write a dialogue you might share with your friend in order to help her understand the issues we've discussed surrounding quantifiers.

"I just don't get it, any of it. It's like, if universal means all, then shouldn't the opposite one mean nothing? It doesn't make sense that the opposite of 'everything is this way' is 'something else exists.' It feels like the opposite should be 'nothing is this way."'

"I can see where you're coming from, and it seems like a lot of the confusion comes from thinking in terms of opposites rather than negation. In math, you don't really need the opposite of a statement to disprove it, you just need the negation. Like, if I were to make a statement with a universal quantifier and say 'all frat boys are jerks"'

"That's not true! David's pretty nice."

"Exactly! You just constructed a negation to my statement without even thinking about it. To prove that I was wrong in saying that all frat boys are jerks, you didn't say 'no frat boys are jerks,' you just said that there exists a frat boy who is not a jerk."

"Well, David's not the only one. There are plenty of decent guys in fraternities. So it's not an existential quantifier, because it's more than one."

"It's still an existential quantifier if you're talking about some instead of one. We use existential quantifiers when we want to talk about at least one, sometimes more than one, but not necessarily everything in the set we're dealing with. I could say 'there exists an integer that is not two' and that would be an existential statement."

"But it's not true. There are a lot of integers that aren't two, not just an integer that is not two. Actually, most integers are not two. Almost all of them are not two."

"That's true, but the point is that at least one integer, but not necessarily all integers, is not two. As long as the statement I'm making isn't claiming anything about the entire set, it's existential instead of universal."

"So would the negation of that statement be 'there is an integer that is equal to two'?"

"Not quite. To negate an existential quantifier, another existential quantifier won't do. There's plenty of room in the integers for things that are and aren't two, so you can't disprove my original assertion by saying that there is an integer that is two. You would need to say that in the set of all integers, there is no two."

"That's false. But I guess it's supposed to be false, because what you said about an integer being not two was true, and to try to disprove it would be false."

"Yes, the negation of a statement is always the opposite truth value of the original statement."

"But why do I have to make a statement about everything in a set to disprove a statement about one thing in the set? Can't I just say that the one thing doesn't have the property that the original statement claimed it did? Before, you said I didn't have to prove that every frat boy was a decent guy, just that there was one who was. Why is that wrong now?"

"To say something about the one thing in the set not having those properties, you kind of need to say something about everything in the set. Let's try another non-math example. If I were to tell you that The Hop sells beef-flavored ice cream, how would you respond?"

"Eew. No it doesn't."

"What you said is the negation of my statement, and it uses a universal quantifier."

"Didn't sound like one."

"No, it didn't, but it will with some re-phrasing. To tell me that The Hop doesn't sell beef-flavored ice cream, you are saying that none of the ice cream that they do sell is beef-flavored. So what you're saying is that in the set of all the flavors sold at The Hop, every flavor shares the quality 'not beef."'

"Leaving no room for your false statement about beef-flavored ice cream existing!"

"That's right! In this case, trying to use an existential quantifier would be like saying 'they sell a lot of flavors that aren't beef,' and that still leaves room for the possibility that they also sell beef. But they don't, because all of the flavors are not beef."

"Thank God for that."

"Amen. So does this whole quantifier thing make more sense now?"

"Yeah, some. I think I'm ready to tackle the homework now."