MATH 280, Section 1
Foundations of Mathematics
Fall 2009

What went on?

The exercise we just completed illustrates a number of important underpinning aspects of mathematics and mathematical proof.

1. **Evidence leads to a statement or "theorem."** Confronted with somewhat sparse data concerning sums of integers (data that could suggest any number of conclusions), we were led to put forth a hypothesis concerning patterns that arise when integers are added to one another.

   This step in the process of mathematical discovery can sometimes be brief one, as it was here. Other times years may be spent in simply trying to figure out what the data before us suggest.

2. **Rudimentary terminology and notation helps to formulate a theorem precisely.** Once we have an understanding of what "should be true," we flesh out our intuition by developing descriptive terminology and precise notation in order to express carefully the mathematical statement we wish to make. In addition to a *conclusion*, the theorem will include *hypotheses*, conditions we stipulate must be true in order to arrive at the conclusion.

   In writing our theorem, appropriately chosen terminology and notation disambiguates our meaning and allows us to express our thoughts to others with the same mathematical notions.

3. **A proof is carried out according to accepted rules of mathematical logic.** Once we have our theorem before us, we seek to demonstrate its truth. More than simply lending someone the "feeling" that a statement is true, we must *prove* the theorem by using *deductive* methods assumed valid by mathematicians and logicians.

   In our case, our proof was *direct*, proceeding directly from the assumptions laid out by the hypotheses and ending with the conclusion. Other sorts of proofs will make use of *contradiction* techniques, *contrapositive* techniques, and more advanced ideas like *induction*.

This class will focus on developing the ideas contained above. We will begin by examining the basic principles of mathematical logic and proof, at the same time using these techniques to investigate properties of sets, functions, and relations.