

*Section 3.4: How to get a nice graph you can feel really proud to put on your refrigerator*

Between precalculus and this course, you've now investigated a good many of the properties that make a function's graph shaped the way it is:

- (1) The location of  $f$ 's graph relative to the axes is determined by its *sign*, and the function's sign changes when  $f(x) = \underline{\hspace{2cm}}$ , that is, at a  $\underline{\hspace{2cm}}$  of the function.
- (2) The slope of  $f$ 's graph is determined by  $f$ 's first  $\underline{\hspace{2cm}}$ , and from the sign of this function we can tell if  $f$  is increasing or  $\underline{\hspace{2cm}}$ . (We can transition from one to the other only at  $\underline{\hspace{2cm}}$  *points* of the function  $f$ .)
- (3) The  $\underline{\hspace{2cm}}$  of  $f$ 's graph is determined by  $f$ 's second derivative, and from the sign of this function we can tell if  $f$  is concave  $\underline{\hspace{2cm}}$  or concave  $\underline{\hspace{2cm}}$ . (We can transition from one to the other only at  $\underline{\hspace{2cm}}$  *points* of the function  $f$ .)
- (4) If  $f(x)$  has a horizontal  $\underline{\hspace{2cm}}$  we can tell what's happening with the function for large (positive and negative) values of  $x$ .
- (5) We can look for points where the function has vertical  $\underline{\hspace{2cm}}$ , often by finding places where we're trying to divide by  $\underline{\hspace{2cm}}$ .
- (6) Finally, other little things tell us about the shape of the graph: does it have *periodic* symmetry (like the common trig functions)? Does it *even* or *odd* symmetry? What's the function's  $y$ -intercept?

Let's do a few examples in which we combine all of these properties to fully understand the function we're given.

**Examples.**

- (1) Analyze  $f(x) = \frac{x^2}{x-1}$  using the ideas above. (Recall that you have critical points wherever the function is not defined!)

(2) Now analyze  $g(x) = \frac{x^2}{1-x^2}$ .

(3) What about  $h(x) = x + \sin(x)$ ? (Find at least a few of its critical points and inflection points.)

(4) One more:  $k(t) = \frac{1}{t} + \sqrt{t}$ .

**Homework.** The following problems are due on *Friday, April 10th*.

- (1) Use the information summarized in this section in order to analyze the functions given below:

(a)  $F(x) = \frac{x^4}{6} - \frac{x^3}{3} + 2x^2$

(b)  $f(x) = x - \frac{1}{x}$

(c)  $g(x) = \sin(2x)$

(d)  $h(x) = e^x + e^{-x}$  (*Hint:* recall that  $e^{-x}$  is just  $\frac{1}{e^x}$ .)

- (2) Find the formula for a function that has precisely the following asymptotes (note that one of them is a horizontal asymptote; the others are vertical!):

$$y = 1, x = -2, x = 0, x = 3, \text{ and } x = 7.$$

*Hint:* rational functions make easy asymptotes...