MATH 191, Section 2
Calculus I
Spring 2009

Section 2.1: The derivative

The most crucial concepts in all of Calc I are ____________ and ______________ . The first of these we'll make more precise later; our goal today is to better understand the second.

Definition. Let f be a function, and let's suppose the independent variable is t (for "time"). The ______________ *of f with respect to t*, denoted $f'(t)$ or $\frac{df}{dt}$, is defined by

$$\lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Notes.

(1) The notation "Δt" can be read "change in t." In fact, any time you see the letter Δ in front of a variable, it's a good chance it's referring to "change in" that variable. For instance, sometimes we'll write Δf to denote the change in f: $\Delta f = f(t + \Delta t) - f(t)$.
(2) The "$\lim_{\Delta t \to 0}$" in front of the quotient is the *limit* with which we'll deal carefully later. For now, just think about Δt being made smaller and smaller and smaller without actually reaching 0.
(3) The letter h is often used to denote Δt, as we did in our discussion on Section 1.3.
(4) As in Section 1.3, the derivative measures the ________________ rate of change of f as t changes, as opposed to the ____________ rate of change of the same function f, found by simply taking a quotient without a limit.

It's worth elaborating on this last point a little bit by considering a few interpretive

Examples.

(1) If f represents the distance an object travels by time t, then $\frac{f(t_2)-f(t_1)}{t_2-t_1}$ gives the *average* rate of change of distance with respect to time on $[t_1, t_2]$...this, as we know, is the *average* ____________ of the object. The derivative $f'(t)$ gives the *instantaneous* ____________ .
(2) If $Q(t)$ represents the amount of electrical charge present in a wire at time t, then $Q'(t)$ (or $\frac{dQ}{dt}$) represents the instantaneous rate at which the charge is flowing through the wire at time t. This quantity is called the ____________ in the wire.
(3) If $s(t)$ represents the probability that a cancer patient survives at least t years after her diagnosis, then $s'(t)$ (or $\frac{ds}{dt}$) represents the instantaneous change in that probability with respect to time.
(4) If $I(p)$ represents the interest level of a reader making her way through Jane Austen's *Northanger Abbey*, where p is the reader's current page number, then $I'(p)$ (or $\frac{dI}{dp}$) represents the rate at which the reader's interest is changing as she reads. How might you interpret a high positive value of I in this case? A negative value?

Computing derivatives. The trick to computing derivatives is, in general, to simplify the quotient as much as you can *before* letting Δt (or h) become 0.

We've already computed $f'(x)$ when $f(x) = x^2$ and when $f(x) = x^3$. Let's do a few more examples, some easier, some harder. For each of these examples you should be able to graph both the original function and its derivative in order to see how they are related to one another.

Examples.

(1) Find $f'(x)$ if $f(x) = x$.

(2) Find $\frac{dg}{dt}$ if $g(t) = 3t^2$.

(3) Find $H'(x)$ if $H(x) = \frac{1}{x}$.

(4) Find $J'(r)$ if $J(r) = \sqrt{r}$. (*Hint*: you may wish to remember that the ____________ of a radical expression $\sqrt{a} - \sqrt{b}$ is given by ____________ . How does this help here?)

(5) One more! Suppose that we understand $f(x)$ and its derivative very well, and we know that $g(x) = f(x) + c$ for some *constant* c. What can we say about the rate of change of g?

Homework (*Due Friday, February 6th*) Do exercises 1, 2, 3, 4, 11, 15, 21, and 27 from Section 2.1 (pages 49–50) of your text.