Section 1.3: Velocity

In precalculus you likely (briefly) studied two sorts of lines that are related to the graph of a function $y = f(x)$:

(1) a ______________ *line* intersects the graph of f in *two* points, while
(2) a ______________ *line* intersects the graphs of f in ___________________ .

The slopes of these lines will often say a good deal about the quantity measured by f.

Example. Suppose that the quantity $f(t)$ gives the distance that an object travels in a given time t.

We all know that *velocity* is found by dividing distance by ______ ; that is, if we want to find the average velocity of the object in question between times t_1 and t_2, we would have to use the following formula:

average velocity $=$ ______________ .

Note that if we plot $f(t)$ on a pair of axes (as you'll do in the space below), the quantity you just wrote down gives the _________ of the ____________ line to the graph of the function $f(t)$:

Example. Suppose that the distance function $f(t)$ is given by $f(t) = t^2$. Find the average velocity of the object on each of the following time intervals: $[1, 4]$, $[1, 3]$, and $[1, 2]$.

In the space below you can draw the graph of $f(t) = t^2$ and indicate the secant lines whose slopes you've just computed:

Example. You can do the same with the distance function $f(t) = t^3$, for the same intervals. Here's some space for your computations and your graph:

That's all well and good if we're looking for an *average* velocity over a period of time, but often we'll be interested in what's happening *right now.* How can we go about computing the so-called ____________________ velocity of an object?

We can *approximate* the instantaneous velocity at a single point in time t by computing the *average* velocity over the time interval $[t, t+h]$ for some very very *very* small quantity h.

Example. Returning to $f(t) = t^2$, find the average velocity of the object over the time interval $[t, t+h]$ for a fixed t.

Got that? Good. Now, to find the instantaneous velocity, let's just suppose that h becomes so close to 0 that it goes away. What value do we obtain for the instantaneous velocity?

Now you can just read off the instantaneous velocity at times $t = 1$, $t = 2$, and $t = 3$:

Notes. Two comments are in order here:

1. What we've just done is computed the slope of the ____________ line to the graph at the value t. The graph you're about to draw will show clearly how the secant lines for smaller and smaller h will approach the tangent line at t:

2. The reason we couldn't have used our "rise over run" method to compute the slope of the *tangent* line is that the denominator in this formula would have been 0, and we all know you can't divide by 0!

Summary. If $f(t)$ gives the distance traveled by an object by time t, its average velocity on the time interval $[a, b]$ is given by

$$\text{average velocity} \; = \frac{f(b) - f(a)}{b - a}.$$

The instantaneous velocity at a time t is given by computing

$$\frac{f(t+h) - f(t)}{(t+h) - t} = \frac{f(t+h) - f(t)}{h}$$

and letting h go to 0.

Note. Formally what we're doing when we find the instantaneous velocity is taking the *limit* of the quotient $\frac{f(t+h)-f(t)}{h}$ as h approaches 0. We'll talk about this in great detail later.

Example. If the distance function of an object is given by $f(t) = t^3$, compute the object's instantaneous velocity at the time $t = 2$.

Homework (*Due Friday, January 30th*) Do exercises 1(a,c,e), 2(a,c,e), 9, 11, 23, and 24 from pages 21–22 of your text.