

Practice Exam 1

This practice exam is similar in length, content, and format to the actual exam. This is not to say that the problems given here represent *all* of the concepts you will encounter on the actual exam, since it's difficult to "cover" all possible subjects in such a short exam! However, if you feel confident on your performance on this practice exam and you've gone over all homeworks and quizzes, you should feel confident about your upcoming performance on the actual exam.

In order to save paper, I have not included space for you to work out your solutions. (The actual exam will provide such space.) Rather, please complete solutions to the below problems on your own paper.

The practice exam is worth a total of 100 points; the point value of each question is provided with that question.

- (1) (16 points) Fill in the blanks in the paragraph below with the most appropriate word or terms.

The derivative of a function $f(x)$, denoted _____, gives the _____ rate of _____ of the function with respect to x . We can also interpret the derivative at a point as the _____ of the _____ line to the function's graph at that point. Formally, computing derivatives requires that we find the _____ the quantity $\frac{f(x+h)-f(x)}{h}$ as h goes to _____, but so far we've just been simplifying this formula and plugging in $h = ______$ when safe to do so.

- (2) (12 points) Using the *definition* of the derivative (and **no** shortcuts!), compute the derivative of the function $f(t) = \frac{2}{t}$.
- (3) (24 points total; 6 points each) Find the derivative of each of the following functions. (You *may* use shortcut formulas!)
- (a) $2\sin(x) - 3\cos(x)$
 - (b) $4e^x - \sqrt{x}$
 - (c) $1000x^2 + 2x^{1000}$
 - (d) $5x^5 + 4x^4 - 3x^3 + 2x^2 - x + 1$
- (4) (10 points) Explain briefly (but carefully) how you can use the derivative of a function f in order to determine the intervals on which f is increasing.
- (5) (20 points total; 10 points each) Let $f(x) = e^x - x$.
- (a) Find the equation of the tangent line to the graph of f at the point $x = 1$.
 - (b) At what values of x is the tangent line to the graph of f *horizontal*?
- (6) (10 points) Find the domain of the function $g(x) = \frac{\sqrt{x+2}}{x^2+2x-3}$.
- (7) (8 points) Suppose we are asked to come up with a function $C(r)$ which gives an estimate for the number of lunar craters with radius r . Which of the following functions is the better choice?

$$C(r) = r^2 \quad \text{or} \quad C(r) = \frac{1}{r^2}$$

Please explain your answer briefly. (*Hint*: think about what the graphs of these functions look like, especially for large value of r .)