MATH 191, Section 2
Calculus I
Spring 2009

A review of functions

Recall that a *function* from one set, A, to another set, B, is really just a rule that tells us how to assign a unique *value* b in B to any given element a in A. We write $b =$ ____ to indicate this assignment.

The set A is usually called the _____ of the function, and B is called the _____ . If f gives the rule defining the function, we can write $f : A \to B$ to indicate the relationship between f, A, and B.

Functions can be described in a number of ways:

(1) verbally, through a description in words
(2) graphically, by simply drawing a picture (this works well for functions from $\mathbb{R}$ to $\mathbb{R}$)
(3) numerically, as a table of numbers
(4) formulaically, giving a formula for the function

We will deal primarily with these last three methods, particularly the fourth.

Example. Sketch a graph of the function defined by $y = -(x-1)^2 - 2$, and state its domain. Also, find the value of the function for each of the following numbers x: $-3, -2, -1, 0, 1, 2, 3$. Also also, state the intervals on which the function is *increasing* and *decreasing*.

So what sorts of functions are there? Here's a list of the sort of functions we might need to deal with:

(1) polynomials (including _____ functions, quadratic functions, *etc.*)
(2) rational functions
(3) absolute value functions
(4) radical (or root) functions
(5) exponential functions
(6) logarithmic functions
(7) trigonometric functions

Let's consider each of these in turn; it'll be up to you to provide a few examples, giving close consideration to one example of each sort.

Polynomials. Here are some examples of polynomial functions:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Rational functions. How is a rational function defined?

Here are some examples of rational functions:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Absolute value functions. How is the plain ol' absolute value function defined? (*Hint*: think "piecewise"!)

Here are some examples of functions involving the absolute value function:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Radical (root) functions. Here are some examples of radical (root) functions:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Exponential functions. How is that an exponential function differs from a polynomial function?

Here are some examples of exponential functions, including at least one example of each "type" of such functions:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Logarithmic functions. How is it that log functions are related to exponential functions?

Here are some examples of logarithmic functions:

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

Trigonometric functions. What are the six basic trig functions?

If we associate an angle θ with a point (x, y) in the plane, how are these functions (of θ) defined in terms of x and y?

Pick one of these to consider more carefully; sketch its graph and give a few of its values, as well as its domain:

A Fundamental Trig Identity. The *Holy Grail* of trig identities is the following fact; *please* remember it!:

$$\sin^2(\theta) + \cos^2(\theta) = \_\_\_\_$$

For *any* value of θ.

Homework (*Due Friday, January 23rd*) For each of the functions listed below, please do each of the following tasks:

(a) Find the domain of the function.
(b) Sketch a graph of the function, labeling your axes and including "tick marks" to give the appropriate scale. (Please try to do this without using a calculator...it might help to remember tricks that allow you to shift and stretch and reflect graphs!)
(c) Give the value of the function at each of the following values of x: $-3, -2, -1, 0, 1, 2, 3$.
(d) Indicate the intervals on which the function is increasing, and the intervals on which the function is decreasing.

Here are the functions:

(1) $f(x) = (x-1)^2 + 3$
(2) $g(x) = \ln(x+2)$ (you don't need to do part (c) for this one!)
(3) $h(x) = \frac{1}{(x+1)^2}$
(4) $k(x) = -2^x + 1$