MATH 191, Section 2
Calculus I
Spring 2009

Final Exam

This exam is worth a total of 200 points, and point values for each question are given below. Please complete this exam **alone**, and without the help of the folks in the Math Lab. You may use your course notes, my on-line notes, and the textbook. Furthermore, you may come and ask me questions, but I promise not to be too helpful.

Please answer every question fully and clearly, on your own paper. Please show your work where appropriate, and circle or box your final answer. If you fail to do any of these things, you *may* not receive full credit.

The exam is due **at 5:00 p.m. on Friday, May 8th**. Late exams will **NOT** be accepted!

1. (32 points total; 8 points each) Compute each of the following limits.

   (a) $\lim_{x\to\infty} \frac{x}{\ln(1+2e^x)}$

   (b) $\lim_{t\to 0^-} \ln(t)$

   (c) $\lim_{x\to\infty} \frac{23x^{101} - 7x^{54} - 307x^{21} + x^{12} - x^9 - 5000x^3 + 1}{704x^{101} + 34x^{91} - 3x^{44} + 2x^{21} - 47x^{11} + 230x - 1}$

   (d) $\lim_{\theta\to\infty} \frac{\tan^{-1}(\theta)}{\theta}$

2. (20 points) A conical tank has a height of 5 meters and a diameter of 2 meters at the top, with its tip pointing downward. Water is entering at a rate of 50 L/min. (Note that there are 1000 liters in a cubic meter.) At what rate is the depth of the water rising when the depth is 1 meter?

3. (32 points total; 8 points each)

   (a) Find $\frac{d}{dx}(\sin(x))$.

   (b) Find $\frac{d}{dx}(\sin(\sin(x)))$.

   (c) Find $\frac{d}{dx}(\sin(\sin(\sin(x))))$.

   (d) If $F_n(x)$ is the function $\sin(\sin(\cdots\sin(x)\cdots))$, where "sin" appears n times, find $F_n'(x)$.

4. (50 points total; 10 points each) Let $f(x) = x(\ln(x))^3$.

   (a) What is the domain of f?

   (b) Find all asymptotes for f.

   (c) Find the intervals on which f is increasing, and the intervals on which f is decreasing. Where are f's extrema, and what types of extrema are they?

(d) Find the intervals on which f is concave up, and the intervals on which f is concave down. Where are f's inflection points?

(e) Use the information you computed above to sketch a rough graph of your function. (Your graph should be consistent with the information above, so it's not enough to plot the graph with a calculator if your information above is wrong!)

5. (30 points total; 10 points each) Calculus comes into play in population biology when we use differentiable functions to model the growth of populations. For instance, suppose we are growing a culture of bacteria in our lab whose population t days after the beginning of our experiment turns out to be given by

$$P(t) = \frac{100e^{t/10}}{t}.$$

(a) Find the population of the bacterial colony, to the nearest bacterium, after 20 days have passed (that is, at time $t = 20$).

(b) Prove carefully that if left unchecked, the population of the colony would grow infinitely large as more and more time passes.

(c) At what time during the colony's growth does the population reach a minimum value? (Be sure to argue why your claim is true using the appropriate calculus techniques.)

6. (20 points) Compute the derivative of the function $g(x) = x + \sqrt{x}$ using the *definition* of the derivative, and ***not*** *shortcut formulas.* (Make sure to show every step clearly!)

7. (16 points) Compute the value of the definite integral $\int_1^2 2x - 1\, dx$ using limits of Riemann sums, and ***not*** *the Fundamental Theorem of Calculus or other shortcuts.* You may use without proof the fact that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$. (*Hint:* set up the Riemann sum carefully using the appropriate intervals and rectangles; a well-drawn picture might help you do this. Then...)