Application 1: Terminal velocity

We've already taken a look at what happens when an object is dropped near the Earth's surface and we neglect air resistance: the acceleration due to gravity is constant ($a(t) = 9.8$ m/s^2), so an object's velocity continues to increase as it falls. As a result, we find that an object falls a distance $s(t) = 4.9t^2$ meters in t seconds.

Things are a little trickier if we take into account the friction on the object due to air resistance. In this case, there is an additional force acting on the object, opposite to the force due to gravity, that tends to slow the object down.

Experiments show that, depending on the shape and size of the object, this force is proportional to the object's velocity. This makes a good deal of sense: the faster the object travels, the more air molecules it encounters, so the more those molecules tend to slow the object down.

The resulting differential equation arises:

$$a(t) = g - bv(t),$$

where $a(t) = v'(t)$ is the object's acceleration, g is the acceleration due to gravity (a constant, 9.8 m/s^2 near the Earth's surface), and b is some positive coefficient describing the object's "aerodynamicity."

1. Show that for any constant C the function $v(t) = \frac{g}{b} - Ce^{-bt}$ is a solution to the differential equation given above.

2. Find the value of the constant C that solves the differential equation subject to the initial conditions $v(0) = 0$. (This corresponds to simply *dropping* the object instead of *throwing* it downward.)

3. The value you got for C now allows you to find the solution $v(t)$ to the given equation with the initial condition $v(0) = 0$:

4. Use a calculator to plot the graph of your solution $v(t)$. Sketch the graph roughly below, being sure to include a scale. What seems to happen for large values of t?

5. Consider the formula for v you gave in (3). What happens to this formula as $t \to \infty$? (Imagine what happens to e^{-bt} for large t, remembering that b is positive...)

   The limiting value you obtain here is called the *terminal velocity* of the object.

6. If the shape of our object remains unchanged as we move it from Earth to Jupiter, where the acceleration due to gravity is $\overline{g} = 23.12$ m/s^2, how does this affect our terminal velocity?