MATH 191, Sections 1 and 3
Calculus I
Fall 2009

Section 3.6: Trig functions

We're in a position to prove a couple of formulas we suspected might be true a while back (simply by considering graphs of the relevant functions):

$$\frac{d}{dx}(\sin(x)) = \cos(x) \quad \text{and} \quad \frac{d}{dx}(\cos(x)) = \underline{\hspace{4cm}} \; .$$

Our verification of these facts will require us to know the following trig identity for sines of sums: for any quantities A and B,

$$\sin(A + B) = \sin(A)\cos(B) + \underline{\hspace{5cm}} \; .$$

(Don't feel the need to memorize this formula; I'll not quiz you on it.) Knowing this, we can use the limit definition of the derivative to compute $(\sin(x))'$ in the space below:

Proving the other formula requires knowing an identity for cosines of sums:

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B).$$

I'll be asking you to verify the cosine derivative formula in the homework, using this identity (see Exercise 52).

Let's use these formulas to help us in the following

Examples.

1. Find $f'(x)$ if $f(x) = x^2 \cos(x)$. What's true about the tangent line at the point $x = 0$?

2. Find the equation of the tangent line to the graph of $y = \sin(x)$ at the value $x = \frac{\pi}{3}$.

3. Find all value of x such that the tangent to the graph of $f(x) = e^x \cos(x)$ is horizontal.

4. Use the Quotient Rule to find $\frac{d}{dx}(\tan(x))$.

As in the last example, the derivatives of the other trig functions are quite easy to come by, using the Quotient Rule:

$$\begin{aligned}
\tfrac{d}{dx}(\cot(x)) &= -\csc^2(x),\\
\tfrac{d}{dx}(\sec(x)) &= \underline{\hspace{6cm}}\ , \text{ and}\\
\tfrac{d}{dx}(\csc(x)) &= -\csc(x)\cot(x).
\end{aligned}$$

I'll ask you to verify the second of these formulas in the homework.

(More) homework from Section 3.6 (pp. 170-171): numbers 5-25 (odds only), 31, 38, 40, and 52. This homework is due on *Friday, October 9th.*