Section 3.5: Higher derivatives

Since the result of differentiating a function $f(x)$ is *another* function (namely, the derivative $f'(x)$), it stands to reason that we can often differentiate *again*, obtaining the derivative *of* the derivative. Often we can do this *again* and *again*, and so on, *ad infinitum*. The resulting derivatives, at each stage, are called the _______________ *derivatives* of the function f, or, singly, the *first*, *second*, _______________ , *derivatives*, and so on.

Notation. Since the first derivative is denoted $f'(x)$, we often denote the second, third, *etc.* derivatives with multiple primes: $f''(x)$, $f'''(x)$, and so on.

For obvious reasons this gets tedious and confusing, so instead we can use "parenthetical" notation:

$$f'(x) = f^{(1)}(x), f''(x) = f^{(2)}(x), f'''(x) = f^{(3)}(x), \ \textit{etc.}$$

If you're a fan of Leibniz notation, here's how you'll write the higher derivatives:

$$f'(x) = \frac{df}{dx}, f''(x) = \frac{d^2f}{dx^2}, f'''(x) = \frac{d^3f}{dx^3}, \ \textit{etc.}$$

The original function can be thought of as the "zeroth" derivative of the function, so sometimes you'll see $f^{(0)}(x) = f(x)$.

Example. Find the first eight derivatives of the function $f(x) = \cos(x)$. Do you see a pattern? Use your pattern to guess the derivative $f^{(73)}(x)$.

Example. Find the first five derivatives of $g(x) = xe^x$. Can you guess what $\frac{d^{15}g}{dx^{15}}$ will be?

Example. Let $s(t)$ indicate the distance an object has traveled by time t.
Recall that the derivative of the distance (or position) function is the ______________ of the object: $v(t) = s'(t)$. The rate at which the ______________ changes with respect to time is known as the ________________________ of the object. Since this is the rate of change *of* the rate of change, it's the ______________ derivative of s:

$$a(t) = \_\_\_\_\_\_\_\_ \quad .$$

If a cantaloup is shot straight upward from a cannon with an initial velocity of 50 m/sec, its height at time t is given by $s(t) = 50t - 4.9t^2$ m. Find the velocity of the melon at times $t = 1$, 2, and 3, and find its acceleration at each of these times.

A ____________________ *equation* is any equation involving a function and one or more of its derivatives. *Solving* such an equation requires that we find a function whose derivatives, when plugged into the given equation, make the equation a valid one.

We're in a position to solve certain of these equations, as long as we know what form of solution to look for.

Example. Let $y = f(x)$, and find two functions which are solutions to the equation $y'' + y' - 6y = 0$ by looking for solutions of the form $y = Ce^{kx}$, for constants C and k. (*Hint:* find the derivatives y' and y'' of this form of function, plug them in, and see what happens...)

Sometimes differential equations come with ______________ *conditions*, stipulations about the values a solution f must have at various times.

Find a function $y = f(x)$ which is a solution to the differential equation in the above example, with the additional initial condition $f(0) = 4$.

What if we demand instead that $f'(0) = 4$?

Homework from Section 3.5 (pp. 164-167): numbers 1-28 (evens only), 34, and 44. This homework is due on *Friday, October 30th.*