MATH 191, SECTIONS 1 AND 3
CALCULUS I
FALL 2009

Section 3.2, part 2: Some more simple derivatives

Before we work on finding derivatives of exponential functions, it's worth taking a moment to understand more deeply the implications of interpreting the derivative as a *function*, rather than simply a number at a particular point.

Example. In the space below, draw the graph of an arbitrary function that is differentiable everywhere, and on the same axes sketch a *rough* graph of the derivative. (*Hint:* it may be easiest to identify first where the derivative is equal to 0, and then use those points to identify intervals where the derivative is positive and negative...)

Example. In the space below, draw the graph of the function $f(x) = |x|$, and on the same axes draw the graph of the derivative. Is the function $f(x)$ differentiable everywhere? Why or why not?

Example. Sketch graphs of $\sin(x)$ and $\cos(x)$ on the same axes, between $x = 0$ and $x = \pi$. Notice anything worth reporting on?

Now, let's get to those exponential functions. A word of warning, first, though:

POWER FUNCTIONS AND EXPONENTIAL FUNCTIONS ARE *VERY* DIFFERENT!

Recall that *power* functions are functions of the form $f(x) = x^n$, and we already know their derivatives ($\frac{d}{dx}x^n =$ ____________). *Exponential* functions, on the other hand, have the form $g(x) = a^x$, for some $a > 0$. To find *their* derivatives we need to return to the definition of $g'(x)$.

Let $g(x) = a^x$. Using the h-definition of the derivative, simplify the formula for $g'(x)$ as much as you can without knowing what a is:

The most important consequence of these computations is the following

Fact. The derivative of an exponential function is proportional to the function itself.

The *constant of proportionality* is the limit above, $\lim_{h\to 0}$ ____________ .

What *is* this limit? Wouldn't it be splendid if we could find a number a that would make this limit 1? Time to...

Play! Use a calculator to estimate the value of $\frac{2^h-1}{h}$ for several small values of h (like $h = 0.1$, $h = 0.01$, and so forth).

Now do the same for $\frac{2.5^h-1}{h}$.

What about $\frac{3^h-1}{h}$?

There seems to be a number somewhere in between 2.5 and 3 such that $\lim_{h\to 0} \frac{a^h-1}{h} = 1$. By using either a calculator or *Mathematica* to pin down even more closely this magical number, you should be able to put an estimate in the space below:

Does this number look familiar? It should! It's nothing other than ___ , the base of the ____________________ ! Summarizing our work on this page, you should be able to write down the corresponding derivative formula:

$$\frac{d}{dx}\left(\underline{\hspace{2em}} \right) = \underline{\hspace{2em}} \quad .$$

One more note (and a couple more examples) before you get to some more homework on derivative rules:

Theorem. If the function f is differentiable at $x = c$, then it is also continuous at $x = c$.

Proof. Since f is differentiable at c, the limit $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists. We need to show that $\lim_{x\to c} f(x) = f(c)$, or, equivalently, $\lim_{x\to c} f(x) - f(c) = \underline{\quad}$.

Here's some space to do just that:

However, it's possible that a function is continuous but not differentiable. We saw one example ($f(x) = |x|$) earlier; here's another

Example. Where is the continuous function $f(x) = x^{1/3} = \sqrt[3]{x}$ not differentiable? Why not? (*Hint:* use the h-definition of the derivative at the problem point, to see just what goes wrong...)

(More) homework from Section 3.2 (pp. 139-142): numbers 21, 22, 35-38, 47, 49, 67, 77, and 87. This homework is due on *Friday, October 2nd.*