Section 3.1: The derivative

The idea of *limit* is the first of three major concepts in calculus; the second is the idea of the *derivative*. Believe it or not, you've already computed a boatload of derivatives, without even knowing it.

Definition. Let f be a function and a a specific value of the independent variable x. The *derivative of f at a*, if it exists, is defined by

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

An equivalent limit, found by letting $h = x - a$ (so that $x = a + h$), is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

If either of these limits exist, we say that f is _____ at $x = a$.

There are many different notations for the derivative of f at a, including

$$f'(a), \frac{df}{dx}|_{x=a}, Df(a), ...$$

The first of these is the one we'll use most often, but the second will come in handy when we start talking about the derivative as a function.

Notice that

the derivative $f'(a)$ is the instantaneous rate of change of f at $x = a$.

Therefore we can also interpret the derivative in a "geometric" way:

the derivative $f'(a)$ is the slope of the _____ line to f's graph at $x = a$.

This makes it easy to find the formula for the tangent line at $x = a$, using the point-slope equation of a line. To get a handle on all of these new definitions, let's just do a bunch of

Examples. *Differentiate* (that is, find the derivative of) each function, at the point indicated.

1. Find $f'(2)$ if $f(x) = \frac{1}{x}$.

2. Find $g'(1)$ if $g(x) = x^2 + 3x$. Also, find the equation of the tangent line to g's graph at the point $x = 1$.

3. Find $h'(4)$ if $h(t) = \sqrt{t}$. Also, find the equation of the tangent line to h's graph at the point $t = 4$.

Just as a point of terminology, the expressions $\frac{f(x)-f(a)}{x-a}$ and $\frac{f(a+h)-f(a)}{h}$ are called _____ *quotients*. Note that just as we approximated instantaneous rates of change by considering average rates of change over short intervals, we can just as easily approximate derivatives by looking at _____ quotients over short intervals.

Homework from Section 3.1 (pp. 124-128): numbers 11, 12, 13, 18, 25, 33, 41, 53, 57, and 69. This homework is due on *Friday, September 25th.*