Section 2.7: The Intermediate Value Theorem (another "picture theorem")

Here's a simple consequence of continuity:

Intermediate Value Theorem (IVT). Suppose that f is ________________ on the interval $[a, b]$. Then if M is any number between $f(a)$ and $f(b)$, there is a number c on $[a, b]$ such that $f(c) =$ ___ .

Think about what this is saying for a moment: as long as our graph has no gaps, jumps, or asymptotes, we'll hit every value that lies between the values at the endpoints.

Here's some room for a picture of the theorem:

The primary usefulness of the IVT, from our point of view, is in proving the existence, and approximating the position, of zeroes of functions that are a bit outside the ordinary.

Idea. Suppose f is a continuous function and $[a, b]$ is an interval on which you believe it has a zero. Then if $f(a) < 0$ and $f(b) > 0$ (or *vice versa*), IVT confirms your suspicions, using $M =$ ___ .

Example. Consider the function $f(x) = \cos(x) + x$. Graphing it (see below) shows that there should be a zero on the interval ________ .

Let's use a calculator to check the value of the function at those endpoints:

$$f(\ \_\_\_\ ) \approx \qquad \text{and} \qquad f(\ \_\_\_\ ) = \qquad .$$

Because one of these is positive and the other is negative, we know that for $M = 0$ there is a c on the interval ________ at which the function must be 0.

We can narrow it down further by dividing our original interval into two *subintervals* of equal length, and testing the value of the function at the midpoint of the original interval: whichever one of the two new intervals we just created gives endpoint values with different signs is guaranteed to have a zero on it!

Which subinterval must have a zero on it, in our example above?

Example. The function $f(x) = x^3 - 2$ has a zero at $\sqrt[3]{2}$, which lies somewhere on the interval $[1, 2]$. Use IVT to narrow down the location of this value.

Homework from Section 2.7 (pp. 106-107): numbers 6, 7, and 15. This homework is due on *Friday, September 25th.*