MATH 191, Sections 1 and 3
Calculus I
Fall 2009

Section 2.5: The algebra of limits

Sometimes we can't simply use a function's continuity to evaluate a limit by plugging in...but we can often do a little bit of algebraic simplification before applying continuity successfully. We've already seen a few examples of this phenomenon. Here's another

Example. Simplify the function in the limit $\lim_{x\to 4} \frac{x^2-16}{x-4}$, and use the simpler form to find the limit.

Notice that we used very crucially the following important fact:

WHEN EVALUATING A LIMIT, YOU DON'T *PLUG IN* THE VALUE YOU APPROACH!

This allowed us to assume that $x \neq 4$, allowing us to cancel the terms $x - 4$ with impunity, using continuity only at the very end.

Here's another example involving algebraic simplification:

Example. Compute the limit $\lim_{\theta\to\pi/2} \frac{\cot(\theta)}{\csc(\theta)}$. (*Hint:* simplify and cancel!)

In both of the cases above, we had to get rid of what are often called ________________ *forms*, which are expressions that look like $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

Below is one more form that arises frequently. You may wish to recall that the ____________ of a radical expression of the form $a + \sqrt{b}$ is $a - \sqrt{b}$. What happens when you multiply these terms together?

Example. Evaluate the limit $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$. (*Hint:* multiply the top and bottom by the right conjugate...)

Here are a few more for you to practice on:

Examples. Find the limits below.

1. $\lim_{x\to 2} \frac{x-2}{x^3-4x}$

2. $\lim_{t\to 0} \frac{e^t-e^{2t}}{1-e^t}$ (*Hint:* what can you factor out of the top?)

3. $\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

4. $\lim_{x\to 2^+} \frac{x}{x-2}$ (*Hint:* don't be so hasty...maybe the limit doesn't exist as a number!)

5. $\lim_{h\to 0} \frac{(3a+h)^2-9a^2}{h}$

Homework from Section 2.5 (pp. 97-98): numbers 3, 7, 10, 13, 16, 19, 24, 27, 39, 47, and 52. This homework is due on *Friday, September 18th.*