## *Section 2.1: Tangents and secants and rates of change*

**Application.** Let's suppose that the function $s(t)$ gives us the distance an object moves as a function of time. (For example, an object falling near the Earth's surface moves according to the function $s(t) = 4.9t^2$ m.)

Our goal? To understand *how fast* the object is moving at a given time $t$.

*Attempt 1: average velocity.* One way to find out how fast the object is moving is to measure how far it's gone at two points in time, say $t_0$ and $t_1$, and use these to compute an __________ *rate of change* of the position of the object. There are two ingredients here:

- the distance the object travels between these two times is given by $\Delta s =$ __________ .
- The period of time during which we are performing our measurement is $\Delta t =$ __________ units long.

What do we do with this information? We're all familiar with finding averages: to find the average number of students per class in three different classes, we add the total number of students together and divide by 3, the number of classes.

Similarly, to find the average __________ we desire (this is the average number of distance units traveled per time unit), we divide:

$$\_\_\_\_\_\_\_\_\_\_ \text{ velocity } = \frac{\Delta s}{\Delta t} = \_\_\_\_\_\_\_\_\_\_ .$$

That's really all there is to it!

**Example.** A grapefruit is dropped from the top of Taipei 101 and falls earthward according to $s(t) = 4.9t^2$ m. (Time is measured in seconds.) What is the average velocity of the grapefruit during the first 2 seconds of its fall? During the time interval $[2, 4]$? The time interval $[4, 6]$?

It will help us tremendously to understand what's happening *graphically.*

Consider the graph of $s(t)$ as a function of $t$. In the space below, you should sketch a graph of $s(t) = 4.9t^2$, corresponding to the example above.

Notice that $\Delta s = s_1 - s_0 = s(t_1) - s(t_0)$ corresponds to the "rise" in our graph from $t = t_0$ to $t = t_1$, while $\Delta t = t_1 - t_0$ corresponds to the graph's " _____ ." Thus in computing the quotient $\frac{\Delta s}{\Delta t}$, we're simply computing the __________ of the _______________ line between the points $(t_0, s_0)$ and $(t_1, s_1)$.

Draw this on the graph above, for $s_0 = 0$ and $s_1 = 2$!

We can summarize our observation thusly:

**Interpretation.** The average rate of change of the quantity $s$ with respect to the quantity $t$ (that is, the average velocity) in the time interval $[t_0, t_1]$ is the __________ of the _______________ line between $(t_0, s_0)$ and $(t_1, s_1)$.

There's really nothing special about velocity here: if we're asked to compute any average rate of change of one quantity with respect to another, the same technique will work.

**Example.** As workers at a new auto factory become more proficient at their tasks, their productivity increases, so that in the first 30 days of the factory's operations the number of cars they produce on a given day $t$ is given by $f(t) = \frac{100}{1+0.5^t}$.

Use a calculator to get a rough sketch of the function $f(t)$, and draw it below:

Now show that there is such a thing as a "learning curve" by comparing the average change in the number of cars produced per day over the period $[0, 2]$ and the same average change over the period $[10, 12]$. (*Hint:* Compute $\frac{\Delta f}{\Delta t}$ with the appropriate values of $t_0$ and $t_1$. Which value is greater?)

Our final comment will motivate our travel in the direction our study now takes us: if we are not simply content with computing *average* rates of change but rather are interested in ________________ rates of change, we may approach our goal by considering secant lines over shorter and shorter time intervals starting at $t = t_0$, as in the graph of $s(t) = 4.9t^2$ below:

In this process, *secant* lines approximate a ____________ line, and the slope of this last line is the one that will determine the ________________ rate of change we desire. That is, to approximate an ________________ rate of change, we can compute various *average* rates of change over smaller and smaller time intervals.

**Homework from Section 2.1 (pp. 66-69):** numbers 3, 4, 8, 9, 13, 19, and 25. This homework is due on *Friday, September 11th.*