MATH 191, Sections 1 and 3
Calculus I
Fall 2009

Section 1.4: Precalculus preliminaries, III (trig functions)

As we indicated on the previous handout, there are six trig functions, two of which are more basic than the others (the remaining four are easily defined in terms of the first two). The functions are as follows:

On the last set of notes we recalled how to obtain these functions from the angle θ inside of a right triangle. We can also obtain them from the *unit circle*.

Recall that if θ is the angle between a point (x, y) on the circle of radius 1 centered at the origin and the positive x-axis, then the point (x, y) is given by ____________________ . You should be able to draw a picture of this below:

Knowing that a full turn around the unit circle corresponds to an angle of 2π (and thus a half-turn is π, a quarter-turn is $\frac{\pi}{2}$, and so forth), this arrangement gives rise to some of the more basic values of $\sin(\theta)$ and $\cos(\theta)$.

Examples. Compute each of the values below.

$\cos(\frac{\pi}{2}) =$ ________________ $\sin(\pi) =$ ________________

$\sin(\frac{\pi}{4}) =$ ________________ $\cos(-\pi) =$ ________________

$\cos(-\frac{3\pi}{4}) =$ ________________ $\sin(\frac{\pi}{3}) =$ ________________

(For the last value, you may wish to recall what happens in a $30 - 60 - 90$ right triangle!)

Just as a reminder, recall that to convert from radians to degrees, multiply by $\frac{180}{\pi}$, and from degrees to radians, multiply by $\frac{\pi}{180}$.

There's other information we can discern from the unit circle: notice that...

1. ...every point on the unit circle has the same x-coordinate as the point on the opposite side of the x-axis from it. That is, $\cos(-\theta) = \cos(\theta)$ for all angles θ. This says that $\cos(\theta)$ is an ________ function!

2. Similarly, the y-coordinate of a point given by $(\cos(\theta), \sin(\theta))$ is the *negative* of the y-coordinate of the point given by $(\cos(-\theta), \sin(-\theta))$. Thus it must be that $\sin(-\theta) = -\sin(\theta)$, and $\sin(\theta)$ is an ________ function!

All of the above information leads to the following graphs for these two functions:

Note that the *periodic* nature of these functions comes from the fact that as we go around the unit circle, the points (x, y) repeat themselves at intervals of length 2π.

Graphing $\tan(\theta)$ and $\sec(\theta)$, for instance, isn't too hard, knowing the above graphs and the fact that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ and $\sec(\theta) = \frac{1}{\sin(\theta)}$. In the space below, sketch rough graphs of $\tan(\theta)$ and $\sec(\theta)$ (where do they have asymptotes?):

Finally, let's talk about identities briefly. Your textbook lists several identities, most of which are relatively rarely used, and I recommend against memorizing them. There is one fundamental identity you should always keep in mind, though:

$$\sin^2(\theta) + \cos^2(\theta) = 1, \text{ for any value } \theta.$$

This identity comes straight from the fact that on the unit circle $(x, y) = (\cos(\theta), \sin(\theta))$, and from the Pythagorean formula.

From this you can derive the following identity, just dividing both sides by $\cos^2(\theta)$:

$$\tan^2(\theta) + 1 = \underline{\hspace{4cm}}, \text{ for any value } \theta.$$

Here's some space to check that this is true:

One more example! Suppose we know that θ lies on the interval $[0, \frac{\pi}{2})$, and that $\tan(\theta) = \frac{2}{7}$. Find each of $\sin(\theta)$, $\sec(\theta)$, and $\cot(\theta)$. (*Hint:* use a right triangle with sides of the appropriate length.)

Homework from Section 1.4 (pp. 31-34): numbers 3, 4, 9, 13, 19, 22, 28, 30, and 31. This homework is due on *Friday, September 4th.*