MATH 191, Sections 1 and 3
Calculus I
Fall 2009

Section 1.3: Precalculus preliminaries, II (classes of functions)

We continue with this brief worksheet, which discusses the different kinds of functions we will encounter on our travels. We'll address these kinds one at a time.

1. *Polynomials.* A *polynomial* in the variable x is any function that has the form

   $$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n,$$

   where we may assume that $a_n \neq 0$.

   Examples. $p(x) = 5x + 6x^4 - x^5$, $q(x) = 1 + x + x^2 + x^3$, and $f(x) = x^{100} - x^{1000}$ are all polynomials.

   To test your memory of the "anatomy" of a polynomial, consider the example $p(x) = 5x + 6x^4 - x^5$ from above and identify each of the following aspects of it:

   - *independent variable*:
   - *coefficients*:
   - *degree*:
   - *leading coefficient*:

   Now give an example of a polynomial with leading coefficient 4 and degree 5:

   What is the domain of a polynomial?

2. *Rational functions.* A *rational function* is any function having the form $\frac{p(x)}{q(x)}$, where p and q are themselves polynomials, and $q(x) \neq 0$.

   Examples. Give three examples of a rational function in the space below.

   What is the domain of the rational function $f(x) = \frac{x^3+1}{x^2-3x+2}$? Why?

Can you describe more generally how to find the domain of an arbitrary rational function?

Find the domain of the rational function $g(t) = \frac{t^7 - 3t + t}{t^4 - t^2}$.

3. *Algebraic functions.* These functions are built up from polynomials and rational functions by including roots and powers of these functions. If a function cannot be built in this fashion, we call it *transcendental.* The domains of all of these more general functions are often harder to compute.

   Examples. The following are all algebraic functions: $f(x) = \sqrt{x^2 + 1}$, $g(s) = \frac{\sqrt[3]{s} - 3s}{(s + \sqrt{s})^2}$, and $v(t) = \sqrt{t + \sqrt{t}}$.

   Come up with two examples of your own algebraic functions.

4. *Exponential functions.* Whereas $f(x) = x^2$ is a polynomial, if we switch the roles of the variable and the constant we obtain an *exponential function.* That is, $g(x) = 2^x$ is an exponential function, as are

$$F(t) = 0.5^t, h(x) = \pi^x, \text{ and } G(x) = e^x.$$

   Recall that the number e is a constant that has special significance we'll discuss in an upcoming section.

   There are three general shapes the graph of an exponential function $f(x) = a^x$ can have, depending on what a is. Draw three graphs in the space below, each illustrating one of the general shapes.

5. *Trigonometric functions.* Recall that the functions $\sin(x)$, $\cos(x)$, $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ are called the *trigonometric functions*, or sometimes, in the context of the unit circle, the *circular functions.*

   On the same set of axes, sketch graphs of both $\sin(x)$ and $\cos(x)$.

   In the space below, draw a right triangle with angle θ, and for each trig function, indicate how to obtain that function using the *adjacent* leg ("adj"), the *opposite* leg ("opp"), and the *hypotenuse* ("hyp") of the triangle.

   Composing to create new functions. Recall that if f and g are functions, then we can define a new function, $f \circ g$, called the *composition* of f with g, by

$$(f \circ g)(x) = f(g(x)).$$

   That is, we first apply g, and then we apply f.

   In the space below, draw an "arrow diagram" illustrating the definition of composition:

Recall that the domain of $f \circ g$ is the set of values x such that

(a) x is in g's domain, and
(b) $g(x)$ is in f's domain.

Example. Suppose $f(x) = 4x + 1$, $g(x) = \sqrt{x}$, and $h(x) = \sin(x)$. Compute each of the following compositions, and give the domain of each.

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(h \circ f)(x)$

(d) $(h \circ h)(x)$

(e) $(f \circ (g \circ h))(x)$

Notice that in general $f \circ g \neq g \circ f$! Please don't fall into this trap!

Homework from Section 1.3 (pp. 23-24): numbers 2, 4, 8, 14, 18, 19, 24, 27, 31, and 33. This homework is due on *Friday, August 28th.*