MATH 191, Sections 1 and 3
Calculus I
Fall 2009

Practice Exam 1

This practice exam is similar in length, content, and format to the actual exam. This is not to say that the problems given here represent *all* of the concepts you will encounter on the actual exam, since it's difficult to "cover" all possible subjects in such a short exam! However, if you feel confident on your performance on this practice exam and you've gone over all homeworks and quizzes, you should feel confident about your upcoming performance on the actual exam.

In order to save paper, I have not included space for you to work out your solutions. (The actual exam will provide such space.) Rather, please complete solutions to the below problems on your own paper.

The practice exam is worth a total of 100 points; the point value of each question is provided with that question.

1. (24 points total; 8 points each) Indicate whether each statement is true or false, and give a *brief* (roughly one- or two-sentence) explanation for your answer.

   (a) The average rate of change of the function $f(x)$ between $x = x_0$ and $x = x_1$ is the same as the slope of the tangent line to f's graph through the point $(x_0, f(x_0))$.

   (b) If $\lim_{x\to c} f(x)$ and $f(c)$ both exist, then the function f is continuous at $x = c$.

   (c) The Intermediate Value Theorem can be applied to the function $f(x) = \sin(x)$ on the interval $[-\frac{\pi}{3}, \frac{\pi}{2}]$.

2. (24 points total; 8 points each) Evaluate each of the following limits. If a limit does not exist (either as a real number or as an infinite quantity), explain briefly why this is the case.

   (a) $\lim_{x\to 0} \frac{\cos(x)}{x+1}$

   (b) $\lim_{t\to -2} \frac{t^2+3t+2}{t+2}$

   (c) $\lim_{\theta\to 0} \frac{\sin(5\theta)}{3\theta}$

3. (8 points) Briefly desribe the purpose of both the Vertical Line Test and the Horizontal Line Test, taking care to describe how the two tests differ.

4. (8 points) Draw the graph of a *single* function f with *all* of the following properties:

   (a) f has a jump discontinuity at $x = 3$,

   (b) both one-sided limits exist for f at $x = 2$, but the ordinary limit $\lim_{x\to 2} f(x)$ does not exist,

   (c) $\lim_{x\to 0} f(x)$ exists but the function is not continuous there, and

(d) $\lim_{x \to -2^+} = \infty$.

5. (12 points total; 6 points each) Suppose that a kumquat is dropped from the top of the Willis (formerly Sears) Tower and that it falls $s(t) = 4.9t^2 - 1.2t$ meters after t seconds have passed.

   (a) Find the average velocity of the kumquat on the time interval $[0, 2]$.

   (b) Find the average velocity of the kumquat on the time interval $[2, 4]$.

6. (10 points total; 5 points each) Let $f(x) = \sqrt{x}$ and $g(x) = x + 1$.

   (a) Find a formula for $f \circ g$ and give its domain.

   (b) Find a formula for $g \circ f$ and give its domain.

7. (14 points total; 7 points each) Let $f(x) = (x - 2)^2 + 1$.

   (a) Sketch a graph of the function f and explain in one sentence how to obtain this graph without using a calculator.

   (b) Explain in one sentence how you can tell that f is one-to-one on the interval $[2, \infty)$, and find its inverse, $f^{-1}(x)$, on that interval.