Application 3: Electrical circuits

In electrical circuitry, *capacitors* (roughly speaking) are electronic components that are used to store a certain amount of electrical charge, called the *capacitance* of the capacitor. To charge a capacitor, one hooks it up to a circuit with an active electrical current; the current in the circuit diminishes once the capacitor is fully charged.

The formula for the current in the circuit as the capacitor is charging is given by the following equation:

$$I(t) = \frac{E}{R} e^{-\frac{t}{cR}},$$

where $I(t)$ is the current in the circuit, and E, R, and c are all (for our purposes) constants which describe the circuit's properties.

Knowing this, we often want to know what the charge contained in the capacitor is at a certain time t while it's being charged.

1. The charge q contained in the capacitor is related to the current I by the formula $I(t) = \frac{dq}{dt}$. Use this information to set up an equation we could solve in order to find q.

2. Suppose we know that some function of the form $q(t) = k(1 - e^{rt})$ will be a solution to the equation you wrote in (1). Differentiate this q and compare the result with the formula you got for $\frac{dq}{dt}$ in (1):

   What does this say that r must be, knowing that the exponents have to match?

3. Now plug the value of r you obtained in (2) back into $q(t) = k(1 - e^{rt})$, and differentiate again. Comparing the result with the formula in (1) once more and matching the coefficient in front of the exponential, compute the value k must be:

4. You can now plug k back into the formula for $q(t)$ given in (2) to obtain the amount of charge contained in the capacitor at time t:

5. Suppose $E = 10$, $R = 1$, and $c = 2$. Use a calculator to graph the function $q(t)$ in this case. What happens to the charge q as t gets very large? What do you suppose is the maximum charge the capacitor will be able to hold?

6. With the same values of E, r, and c as in (5), find the length of time it will take for the charge on the capacitor to reach *half* of its maximal value.