Graph Theory, Exercise Set 1

Working together in whatever way you find most useful, beneficial, and efficient, please try to solve as many of the following graph-theoretic exercises as you can before we meet again tomorrow. Please be prepared to present your solutions and findings to the group when we next meet.

1. Track down the definition of a *graph homomorphism* $f : G \to G'$, $G = (V, E, \phi)$, $G' = (V', E', \phi')$. (If the definition you find is cast in different notation or terminology, which very well might be the case, do what you can to put it into our notation and terminology!) Give a few examples of graph homomorphisms, enough to illustrate the principle.

2. Do the same as in Exercise 1, only now for a *graph isomorphism*. (This should be easy once you've done Exercise 1!)

3. Find a definition for the *degree sequence* of a finite graph G, and put it into our notation and terminology. Give a few examples. What can you say about the degree sequences of G and G' if there is a homomorphism from G to G'? What if there is an *iso*morphism from G to G'?

4. Recall that a graph G is called *planar* if G can be drawn in the plane in such a manner that edges do not cross elsewhere than vertices. The geometric realization of a planar graph gives rise to regions in the plane called *faces*; if G is a finite planar graph, there will be one *unbounded* (*i.e.*, infinite) face, and all other faces (if there are any) will be *bounded*.

   Given a planar realization of the graph G, let $v = |V|$, $e = |E|$, and let f be the number of faces (including the unbounded face) of G's realization. Prove that the *Euler formula*

   $$v - e + f = 2$$

   must hold for a *connected* planar graph.

5. Compute the value of $v - e + f$ for the geometric realization of K_5 on the torus we came up with in seminar, by "cutting open" the torus and laying it flat. (*Hint*: be very careful about which regions in the "cut open" torus really belong to the same face.)