Geometry, Exercise Set 1

Working together in whatever way you find most useful, beneficial, and efficient, please try to solve as many of the following graph-theoretic exercises as you can before we meet again tomorrow. Please be prepared to present your solutions and findings to the group when we next meet.

1. Describe at least two *different* metrics for the set C consisting of the unit circle $\{(x, y) \in \mathbb{R}^2 \mid \sqrt{x^2 + y^2} = 1\}$. (*Hint*: for one of them, think about a metric on a space in which the circle lies. For the other, imagine what would happen if your whole world were the circle.)

2. Let D be the *unit disk* $\{(x, y) \in \mathbb{R}^2 \mid \sqrt{x^2 + y^2} \leq 1\}$. Define $\rho : D \times D \to \mathbb{R}$ by letting ρ use the standard Euclidean metric for points (x_1, y_1) and (x_2, y_2) on the *same radius* from the origin, and

$$\rho\big((x_1, y_1), (x_2, y_2)\big) = \sqrt{x_1^2 + y_1^2} + \sqrt{x_2^2 + y_2^2}$$

otherwise. Prove that $(\mathbb{R}^2, \rho)$ is a metric space. ($(\mathbb{R}^2, \rho)$ is called the *French Railroad Space*.)

3. Describe at least three *different* metrics for the set $\mathbb{R}^3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$. (You do not have to *prove* that they are metrics; merely giving a "hand-waving" proof in each case will do.) Can you generalize your constructions to $\mathbb{R}^n$, n any positive integer n?

4. Why does $\hat{\rho} : \mathbb{R}^2 \to \mathbb{R}$ defined by $\hat{\rho}\big((x_1, y_1), (x_2, y_2)\big) = |x_2 - x_1|$ fail to give a metric?