Final Symposium

The following is the schedule of talks for Friday, July 31st.

1:00 – 1:15 Matthew Weiser: *Using cellular automata to generate variations of the Sierpiński triangle with overlaps*
1:20 – 1:35 April Scudere: *Catalan numbers and random trees*
1:40 – 1:55 Chelsie Batten: *Asymptotic connectivity of hyperbolic graphs*
2:00 – 2:15 Nathan Salazar: *Infinite families with symmetric unimodal independence polynomials*

2:15 – 2:25 Break

2:25 – 2:40 Michael James: *Finding a lower bound on the number of graceful labeling of r-regular caterpillars*
2:45 – 3:00 Cara Newcomb: *Ramsey numbers of bistars and caterpillars*
3:05 – 3:20 Cindy Cook: *Using Markov theory to approximate the expected diameter of random graphs*
3:25 – 3:40 Noah Pang: *Graph polynomials of Cayley graphs*

Abstracts

Chelsie Batten *Asymptotic connectivity of hyperbolic graphs*

Asymptotic connectivity is a measure that offers a description of the degree to which an infinite graph is interconnected. In this talk we will investigate the asymptotic connectivity of regular triangulations of the hyperbolic plane. We will prove that the limiting asymptotic connectivity of these hyperbolic graphs is 3 by analyzing the various types of vertices found in these graphs and determining their contributions to the total connectivity. This talk will give the first proof of the existence of the asymptotic connectivities of an infinite family of hyperbolic graphs. Some knowledge of hyperbolic geometry is assumed.

Cindy Cook *Using Markov theory to approximate the expected diameter of random graphs*

We will discuss particular random trees constructed by a process called Use It or Lose It. This process gives rise to infinitely many nonisomorphic trees, which we will classify into ten states. We will prove the different structures these particular trees can form and their transitions from one state to the next. By finding the transition probabilities of each tree, we can use Markov theory to approximate the probability we are in a given state at time t allowing us to find an approximation for the expected diameter at time t.

Michael James *Finding a lower bound on the number of graceful labeling of r-regular caterpillars*

Work has been done on calculating the number of graceful labelings of paths, but what about slightly more complex graphs? The next most simple class of trees is caterpillars, more specifically, r-regular caterpillars, where each vertex in the caterpillar's spine has r leaves attached to it. In this talk we will obtain a lower bound on the number of graceful labelings of these types of graphs. We will show how to combine two such graphs to generate larger and larger gracefully labeled graphs which we can then count by employing an inductive proof.

Cara Newcomb *Ramsey numbers of bistars and caterpillars*

A *caterpillar* is a tree consisting of a central path, from which each vertex has a given number of leaves. Of particular interest are *bistars*, caterpillars which have only 2 vertices as the central path. We will examine the Ramsey number of specific examples, as well as generalized bistars, where the Ramsey number is the smallest integer n such that any 2-coloring of K_n contains a monochromatic copy of the bistar. We will then utilize similar techniques to establish lower bounds of Ramsey numbers of caterpillars.

A basic familiarity with Graph theory is needed.

Noah Pang *Graph polynomials of Cayley graphs*

Graph polynomials such as the chromatic polynomial can determine certain structural properties of the graphs they arise from. The Cayley graph of a group is a labeled graph used to represent a group. The purpose of this paper is to analyze the graph polynomials of Cayley graphs and to determine conditions under which a graph polynomial of a Cayley graph will determine the group the graph arose from. Some knowledge of basic group theory and graph theory is assumed.

Nathan Salazar *Infinite families with symmetric unimodal independence polynomials*

The independence polynomial of a graph is a way of listing the number of independent sets of different sizes that exist in the graph. In this talk, we derive a recurrence relation for calculating the independence polynomial of a certain family of graphs, and then use an inductive proof to show that these polynomials will be symmetric unimodal. That is, the coefficients form a sequence, $a_1, ..., a_k$, with a single peak at $a_{k/2}$, and are symmetric about this point. We also define an iterative process that will produce infinite families of graphs with the same property.

April Scudere *Catalan numbers and random trees*

The Catalan numbers are a well-known integer sequence that arises in a variety of combinatorics problems. This talk will explore the occurrence of these numbers in a particular random tree construction created by Bahls, Knox, and McClure. They conjectured that the Catalan numbers appear as the number of recurrent states in a complex system associated with the construction. We will describe the basic tree construction and prove that the Catalan numbers give us the number of recurrent states. The proof will involve an inductive argument on the rows of Catalan's triangle.

Matthew Weiser *Using cellular automata to generate variations of the Sierpiński triangle with overlaps*

It is well known that Pascal's triangle can be generated with a cellular automaton (CA) construction. Moreover, taking cell values mod 2 yields an approximation of the Sierpiński triangle. It is then natural to ask: What other fractal objects may be represented by cellular automata? Specifically, given a variation of the Sierpiński triangle's iterated function system which includes significant overlaps, can we identify a CA which describes it? We will describe a specific family of fractals which are related to the Sierpinski triangle and which include overlap, and then give a representative cellular automaton construction, with an inductive proof of equivalence.