Review questions for Geometry Quam 2

Monday, September 28

Quam 2 will be this Friday, October 2; this review sheet is to help you prepare.

1. State the three axioms of incidence.
   This exact problem will *definitely* be on the quam.

2. Prove the following assertions in incidence geometry.

   (a) If ℓ_1 and ℓ_2 are distance lines that are not parallel, then ℓ_1 and ℓ_2 intersect at precisely one point.
   (b) There exist at least three distinct lines.
   (c) Given any line ℓ, there is at least one point not on ℓ.
   (d) Given any point P, there are at least two lines through P.

   None of these proofs will be on the quam but there will be one problem that is right at this same level of difficulty.

3. State Playfair's axiom.
   This exact problem will *definitely* be on the quam.

4. Consider the following possible nominations for sets of "points" and "lines".

   (a) Set of points: $S = \{A, B\}$; lines: two element subsets of S.
   (b) Set of points: $S = \{A, B, C\}$; lines: two element subsets of S.
   (c) Set of points: $S = \{A, B, C, D\}$; lines: two element subsets of S.
   (d) Set of points: $S = \{A, B, C, D, E\}$; lines: two element subsets of S.
   (e) Set of points: $S = \{A, B, C, D, E\}$; lines: three element subsets of S.

   Which of these are models of incidence geometry? Which of these are models of incidence geometry plus Playfair's axioms? In both cases, for those that are not models, state exactly which axioms are violated and why.
   A subset of this problem will *definitely* be on the quam. There might be a minor modification as well.

5. Show that Playfair's axiom is independent of incidence geometry.

6. Recall that an affine plane is a model of incidence geometry satisfying the following stronger form of Playfair's axiom: For every line l and every point P, there is a unique line m containing P and parallel to l. Now suppose that a particular model of affine geometry has the additional property that every line has at least three points on it. What is the smallest number of points this model can have? Justify your assertion.

7. State the first three axioms of betweenness.