Some more proofy problems and a solution.

October 23, 2009

We've got a somewhat harder quam coming up next Friday, October 30. There *will* be some proof oriented problems on the quam. Here are a few more example type problems to help you practice, together with a solution to one of the problems we played with the other day. It is *essential* that you spend some time with these problems. You *cannot* expect to learn how to discover proofs by coming to class alone. This skill is only developed through your hard work. We will discuss these in class on Monday and possibly Wednesday.

Three new problems

These problems are all meant to be approached in the context of neutral geometry, i.e. using the axioms we've developed to this point. In particular, there is no parallel postulate.

1. Suppose that $\overline{AB} \cong \overline{CD}$ and that E is a point between A and B. Show that there is a unique point F between C and D such that $\overline{AE} \cong \overline{CF}$.

2. Given $\triangle ABC$ and $\overline{DE}$ such that $\overline{DE} \cong \overline{AB}$, show that on a given side of $\overleftrightarrow{DE}$ there is a unique point F such that $\overline{DF} \cong \overline{AC}$ and $\overline{EF} \cong \overline{BC}$.

3. For every line ℓ and for every point P not on ℓ, show there is a line through P and perpendicular to ℓ.

A recently considered problem

This is problem 9.2 from the text: Suppose that $\overrightarrow{AD}$ is in the interior of $\angle BAC$ and that $\overrightarrow{AE}$ is in the interior of $\angle DAC$, as shown in figure 1. Show that $\overrightarrow{AE}$ is in the interior of $\angle BAC$.

Let P be on $\overrightarrow{AE}$ but not equal to A. We must show that P is in the interior of $\angle BAC$ or, equivalently, that

- P is on the same side of $\overleftrightarrow{AB}$ as C and
- P is on the same side of $\overleftrightarrow{AC}$ as B.

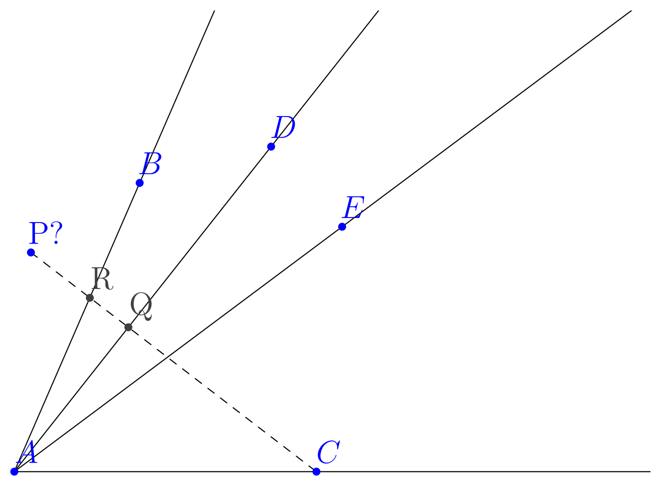

Figure 1: For the proof of exercise 9.2

We will demonstrate the first of these items here, leaving the second as a further exercise. The assumption that P is in the interior of $\angle DAC$ yield, in particular, that P is on the same side of $\overleftrightarrow{DA}$ as C. Suppose for contradiction that P is *not* on the same side of $\overleftrightarrow{AB}$ as C. Then, either

1. P is on $\overleftrightarrow{AB}$ or
2. P is on the opposite side of $\overleftrightarrow{AB}$ as C, as shown by the dashed line in figure 1.

Case 1: P is on $\overleftrightarrow{AB}$. In this case, consider the segment $\overline{PC}$. By the crossbar theorem, this intersects $\overleftrightarrow{AD}$ at a point Q. This contradicts the fact that P is on the same side of $\overleftrightarrow{AD}$ as C.

Case 2: P is on the opposite side of $\overleftrightarrow{AB}$ as C. By definition, this means that there is a point R on $\overleftrightarrow{AB}$ such that $P * R * C$. Now apply the crossbar theorem to $\overline{RC}$ to obtain Q on $\overleftrightarrow{AD}$ such that $R * Q * C$. Of course, $P * R * C$ and $R * Q * C$ imply that $P * Q * C$ by a previous homework problem. Thus, again, $\overline{PC}$ intersects $\overleftrightarrow{AD}$, which is a contradiction.