A few fractal problems

1. Here are a few properties that we might hope the notion of dimension would have.

   (a) If f is a similarity and E is a set, then $\dim(f(E)) = \dim(E)$.

   (b) If $E \supset F$, then $\dim(E) \geq \dim(F)$.

   (c) If E may be decomposed into countably many pieces,

   $$E = \bigcup_{i=1}^{\infty} E_n,$$

   then $\dim(E) = \sup_n (\dim(E_n))$

   Explain what's going on in these statements and why they might be desirable properties for dimension.

2. Suppose a digraph fractal pair has the digraph shown in figure 1 (a).

   (a) Show that both digraph fractals can be generated by a single IFS.

   (b) Suppose that each edge in the digraph corresponds to a similarity with contraction ratio 1/3. Compute the digraph fractal dimension of the sets.

3. We typically like our digraphs to be *strongly connected*; i.e. there should be a path from any given vertex to any other given vertex.

   (a) Assuming a digraph is strongly connected, show that any associated digraph IFS of similarities produces a list of fractals that all have the same dimension.

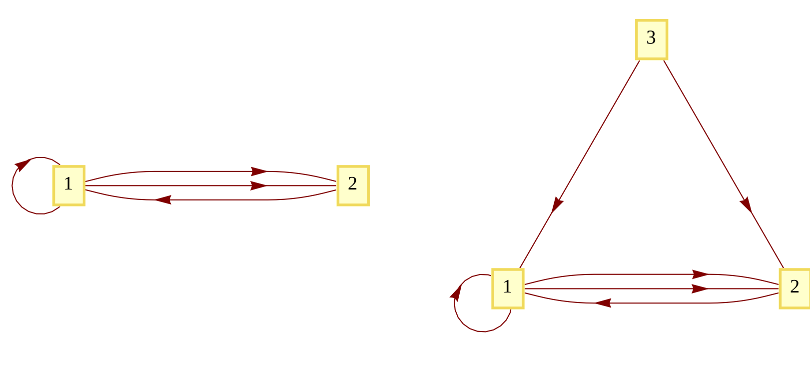
   (b) Suppose that a triple of digraph self-similar sets has the digraph shown in figure 1 (a).

4. Compute the fractal dimension of the z-curve, shown in the bottom right of our IFS practice sheet, using the formula $r_1^s + \cdots + r_m^s = 1$.

5. Let $\{f_1, f_2\}$ be the IFS on the line with $f_1(x) = \frac{1}{2}x$ and $f_2(x) = \frac{1}{4}x + \frac{3}{4}$.

   (a) Compute the dimension of the attractor using the formula $r_1^s + \cdots + r_m^s = 1$.

   (b) Compute the dimension of the attractor using the definition of box-counting dimension.



(a) (b)

Figure 1: A couple of digraphs.